

# A systematic approach to constructing tax-aware portfolios

- Interest income from bonds is typically taxed at much higher rates than qualified dividends from equities. For this reason, advisors often propose that taxable investors use a tax-efficient portfolio that substitutes tax-free, municipal bonds for the entire fixed income sleeve. Although it is more tax-efficient, this type of substitution portfolio has less diversification because of the additional credit risk of municipal bonds. This paper investigates whether replacing the entire taxable fixed income allocation with municipal bonds is optimal for generating after-tax wealth.
- According to Vanguard's portfolio construction framework, portfolios should be constructed based on an investor's goals and preferences. This paper proposes a methodology to construct tax-efficient multiasset portfolios with the goal of maximizing the expected utility of after-tax wealth, by using an enhanced tax-aware version of the Vanguard Asset Allocation Model (VAAM). The tax-aware optimized portfolios enable the best of both worlds—diversification and tax efficiency.
- Our findings show that optimal tax-efficient portfolios should include an allocation to taxable bonds, as they enable portfolio diversification. Our new tax-aware methodology can be tailored to specific client assumptions, including whether investors draw all, some, or none of their income from the tax-efficient portfolio, and whether they plan on liquidating their portfolio at the end of the investment period.

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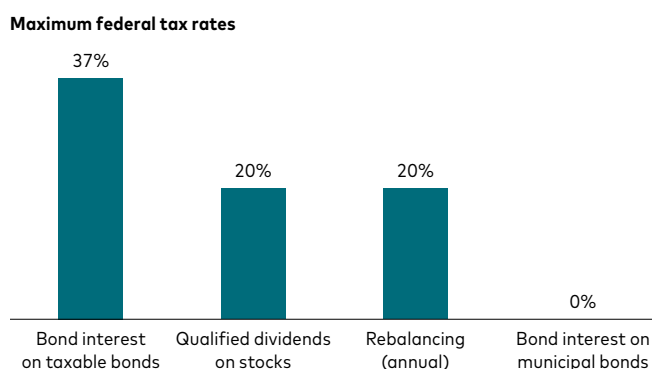


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## Introduction

Just as short-term capital gains tax rates are much higher than long-term capital gains rates, taxes on bond interest are much greater than on qualified dividends from equities, as illustrated in **Figure 1**. More specifically, at the federal level, interest on taxable bonds is taxed at ordinary income tax rates, which are currently as high as 37% for investors in the highest income bracket. In contrast, qualified dividends for equities and long-term capital gains from annual portfolio rebalancing are taxed at much lower rates of 0%, 15%, or 20%, depending on the investor's income bracket.

**FIGURE 1.**  
**Taxes on bond interest, qualified dividends, and portfolio rebalancing**



**Notes:** Dividends can be "qualified" for special tax treatment. (Those that aren't are called "nonqualified.") Most payments from the common stock of U.S. corporations are qualified as long as the investment is held for more than 60 days. Stocks of foreign companies traded through American Depositary Receipts (ADRs) or on U.S. markets may also be qualified. For dividends passed through by a fund to be qualified, the fund must first meet the more-than-60-days requirement for the individual securities paying the dividends. Additionally, the owner of the fund must own the fund shares for more than 60 days. Annual rebalancing can trigger the selling of assets that have appreciated and the paying of capital gains taxes.

**Sources:** Vanguard and the IRS, based on 2022 tax rates.

Given these large tax rate differentials, the industry norm has been to substitute municipal bonds for the entire bond portion of the portfolio, for investors seeking a tax-efficient portfolio within their taxable accounts. This simple mental accounting considers only one dimension, the tax inefficiency of taxable bonds versus the tax-free treatment of municipal bonds. However, this ad hoc approach de-emphasizes diversification, as it doesn't account for the additional credit risk that municipal bonds carry relative to the broad bond market. Additionally, this approach doesn't address the needs of investors in different income tax brackets. In other words, is such a substitution portfolio optimal for investors in all tax brackets?

According to Vanguard's portfolio construction framework (Aliaga-Díaz et al., 2022), portfolios should be constructed based on an investor's goals and preferences. More specifically, this paper proposes a methodology to construct tax-efficient multiasset portfolios with the goal of maximizing the expected utility of after-tax wealth by using an enhanced tax-aware version of the Vanguard Asset Allocation Model (VAAM), our portfolio construction engine. It optimizes asset allocation based on asset returns, correlations, and volatility distributions of the Vanguard Capital Markets Model® (VCMM) and incorporates tax considerations for each asset class.

**IMPORTANT:** The projections and other information generated by the VCMM regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. Distribution of return outcomes from VCMM are derived from 10,000 simulations for each modeled asset class. Simulations as of June 30, 2022. Results from the model may vary with each use and over time. For more information, see Appendix B.

This paper investigates whether replacing the entire taxable fixed income allocation with municipal bonds is optimal for generating after-tax wealth, as our tax-aware portfolio construction methodology targets the best of both worlds—diversification and tax efficiency. We find that the allocations are sensitive to assumptions such as (1) an investor's tax bracket, (2) whether investors draw all, some, or none of their income from the tax-efficient portfolio, (3) whether investors plan to liquidate their portfolio at the end of the horizon, and (4) market conditions and expected returns. In other words, the methodology can enable customized recommendations based on client personas.

### Methodology

The tax-aware portfolio construction methodology builds on the VAAM (Aliaga-Díaz et al., 2019), an expected-utility-based model that assesses risk and return trade-offs of various portfolio combinations based on user-provided inputs such as risk preference,

investment horizon, and the asset classes that are included. The VAAM optimizes portfolios to be total-return-efficient, but so far it has been tax agnostic.

Functionality has been added to allow for the evaluation of portfolios on an after-tax basis, as illustrated in **Figure 2**. We use the same VCMM return distributions that are used by the traditional VAAM, but we break the total returns into their price and income returns. While the existing VAAM methodology is designed to maximize utility of wealth, the enhanced methodology adds the capability to do so on an after-tax basis. With this additional information, the taxes described above can be applied to stock dividends, bond interest, and any applicable capital gains taxes resulting from portfolio rebalancing, as illustrated in Figure 1. The enhanced VAAM also tracks each asset's weighted average cost basis through time to allow for the offsetting of gains and losses each year as the portfolio is rebalanced back to the target allocation.

**FIGURE 2.**  
**Tax methodology assumptions**

Bond interest	Equity qualified dividends	Rebalancing	Portfolio is only source of income?	Portfolio liquidated after 10 years?
<ul style="list-style-type: none"> <li>• Taxed at higher ordinary income rates</li> <li>• Federal rates as high as 37%</li> <li>• Based on income level</li> <li>• VCMM: Income and price return</li> </ul>	<ul style="list-style-type: none"> <li>• Taxed at lower long-term capital gains rate</li> <li>• Federal rates of 0% / 15% / 20%</li> <li>• Based on income level</li> <li>• VCMM: Income and price return</li> </ul>	<ul style="list-style-type: none"> <li>• Taxed at lower long-term capital gains rate for annual rebalancing</li> <li>• Federal rates of 0% / 15% / 20%</li> <li>• Based on income level</li> <li>• Weighted average cost basis</li> </ul>	<ul style="list-style-type: none"> <li>• If so, a larger allocation to taxable bonds is appropriate</li> <li>• If other sources of income: Marginal bracket-based allocation</li> </ul>	<ul style="list-style-type: none"> <li>• Affects optimal allocation</li> </ul>

Source: Vanguard.

The new methodology is flexible enough to consider varying client situations. For example, there is a feature that enables the portfolio to be rebalanced using annual income interchangeably across assets to mitigate potential rebalancing costs. In addition, there is the ability to specify whether the portfolio is to remain invested or liquidated at the end of the period; if it is liquidated, capital gains tax would be applied to the ending-period cumulative unrealized gains. Finally, there is the ability to model any combination of income, dividend, and capital gains tax rates that might be appropriate for when federal and state tax rates change or for building customized portfolios for specific clients.

This enhanced methodology adds capabilities to consider taxes for constructing portfolios within taxable accounts not currently available in the industry. While the traditional method of using municipal bonds in taxable accounts considers only the preferential tax treatment of this asset

class, the VAAM also considers the additional credit risk and volatility municipal bonds add to a portfolio. It considers the overall stock and bond mix in the portfolio allocation and the mix of municipal and taxable bonds simultaneously to optimize the portfolio.

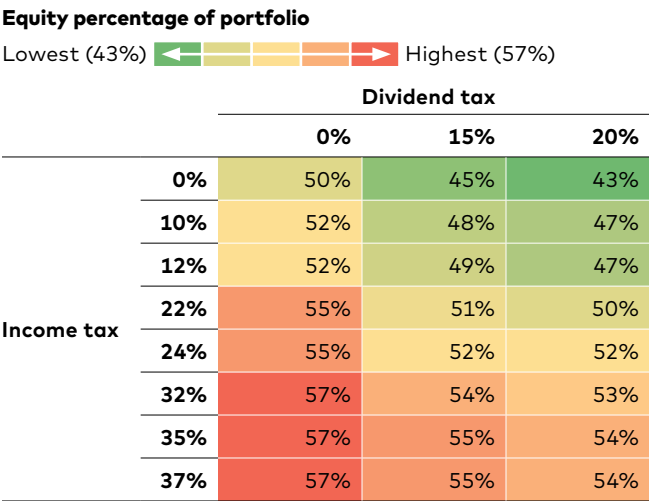
In addition, it can use initial conditions from the VCMM to create time-varying portfolios, which better reflect expectations for after-tax asset class returns over the medium term (the next 10 years). This innovative combination of features and capabilities allows for the construction of tax-aware portfolios that are well diversified yet tax efficient. Further research will include considerations for the alternative minimum tax, treatment of nonqualified dividends for equities, and rebalancing based on tax lots, which we have omitted for the sake of simplicity in this analysis.

How optimal allocations can change based on tax rates

The tax-aware VAAM was evaluated with a wide range of asset classes and numerous tax regimes to verify that the results produced portfolios that were in line with expectations. In the analysis below, we applied flat tax rates (for income, dividends, and capital gains) to the entire portfolio based on the current IRS tax brackets. In addition, these portfolios were calibrated to 50% stocks and 50% bonds before taxes to be able to identify clearly how taxes affect the optimal portfolio allocation.

In **Figure 3**, we show the total equity allocation for a typical four-asset-class portfolio consisting of U.S. and non-U.S. stocks and U.S. and non-U.S. taxable bonds. First, the 50/50 portfolio (pre-tax) demonstrates the trade-offs between stocks and taxable bonds after accounting for the disproportionate tax treatment of the two asset classes, up to 20% for stock dividends and up to 37% for taxable bond income. As the income tax rate increases, causing a higher tax drag on bond interest, so too does the total portfolio equity allocation, which increases from 50% in the pre-tax portfolio to 57%. Consequently, as the dividend tax rate increases, the equity allocation decreases for each income tax bracket. These findings align with conventional wisdom on the after-tax treatment of stocks and taxable bonds.

FIGURE 3. Total equity allocation of a four-asset-class portfolio for various tax rate combinations




Notes: Portfolios are constructed based on 10-year VCMM asset class forecasts, as of June 30, 2022. See Appendix B for more information on the VCMM.

Source: Vanguard.

Next, municipal bonds were added to the previous portfolio, to evaluate the effect that tax-free interest income has on the portfolio. **Figure 4** again shows how the total equity allocation changes based on both the income tax level and the dividend tax rate, but now for a portfolio with five asset classes. As with the previous portfolio, the equity allocation increases as the income tax rates increase and decreases as the dividend tax rates increase. However, with the addition of municipal bonds, the overall asset allocations are a bit closer to the target of 50/50 than for portfolios that did not include them.

Given the tax-free treatment of municipal bonds (at the federal level), intuition would suggest that the portfolios should maintain a more stable asset allocation mix of stocks and bonds and that municipal bonds should replace taxable bonds for higher income tax brackets.

**FIGURE 4.**  
**Total equity allocation of a five-asset-class portfolio for various tax rate combinations**

**Equity percentage of portfolio**  
Lowest (43%)  Highest (55%)


		Dividend tax		
		0%	15%	20%
Income tax	0%	50%	46%	43%
	10%	51%	47%	44%
	12%	52%	48%	45%
	22%	53%	48%	46%
	24%	53%	49%	47%
	32%	54%	49%	48%
	35%	55%	50%	49%
	37%	55%	51%	50%

**Notes:** Portfolios are constructed based on 10-year VCMM asset class forecasts, as of June 30, 2022. See Appendix B for more information on the VCMM.

**Source:** Vanguard.

**Figure 5** displays the allocation to municipal bonds as a percentage of the total bond allocation. Municipal bonds replace most of the bond allocation for the highest income tax brackets (over 32%), which is expected given the tax-free treatment of their income at the federal level. However, an allocation to taxable bonds persists, because of the interest rate environment. At lower tax brackets, municipal bonds replace taxable bonds to a much lesser degree, as expected. For these lower tax brackets, the relatively lower drag of taxable bonds justifies a higher allocation to them than with higher tax brackets.

**FIGURE 5.**  
**Municipal bonds as a percentage of the total bond allocation for various tax rate combinations**

**Municipal bonds percentage of fixed income**  
Lowest (54%)  Highest (91%)

		Dividend tax		
		0%	15%	20%
Income tax	0%	58%	56%	54%
	10%	78%	77%	75%
	12%	79%	77%	76%
	22%	83%	79%	78%
	24%	85%	82%	81%
	32%	87%	86%	85%
	35%	89%	88%	86%
	37%	91%	90%	88%

**Notes:** Portfolios are constructed based on 10-year VCMM asset class forecasts, as of June 30, 2022. See Appendix B for more information on the VCMM.

**Source:** Vanguard.

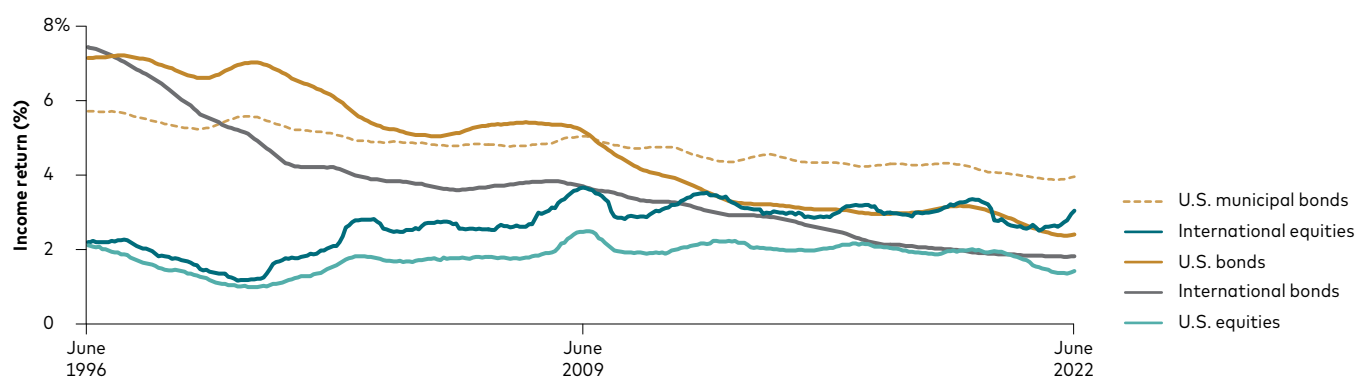
## A time-varying approach

Markets and economic conditions change, and these changes can affect the outcome of an investor's portfolio (Wallick et al., 2020). This is especially true for portfolios that have tax efficiency as an objective because the after-tax returns of an asset class are largely affected by the level of income that it generates. **Figure 6** shows the historical trailing 12-month income

returns for the five asset classes we used in our testing. The U.S. and international equity income returns dropped from 1996 until early 2001, and then rose again, staying in a tight range. Bond income returns, on the other hand, were on a gradual decline for the entire period. The implication is that over this period the high tax burden that bond income carries was also on the decline.<sup>1</sup>

**FIGURE 6.**  
**Income returns over time**

12-month income returns



**Note:** U.S. equities are represented by the MSCI USA IMI Index; international equities are represented by the MSCI AC World ex USA IMI Index; U.S. bonds are represented by the Bloomberg U.S. Aggregate Bond Index; international bonds are represented by the Bloomberg Global Aggregate ex-USD Bond Index; and municipal bonds are represented by the Bloomberg Municipal Bond Index.

**Source:** FactSet.

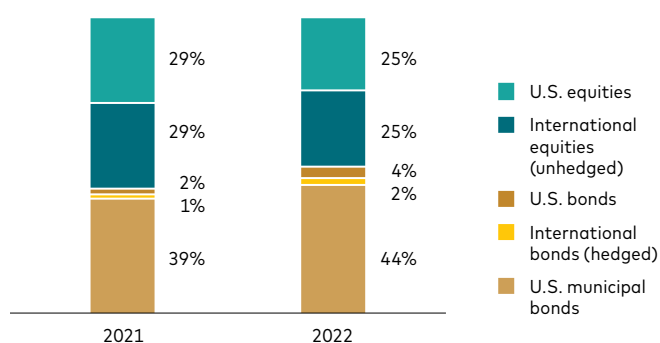
<sup>1</sup> Income tax rates also changed during this period.

Over the medium term, the changing yields and valuations can have a meaningful impact on the expected income and total returns, which in turn can lead to investors having portfolios that do not meet their objectives or achieve their investment goals. For this reason, our tax-aware VAAM methodology lends itself to following a time-varying asset allocation approach. Portfolio asset allocations can be systematically adjusted away from allocations that would otherwise jeopardize an investor's chance of achieving investment success.

**Figure 7** compares an after-tax VAAM-derived portfolio based on 10-year VCMM forecasts as of June 30, 2021, with a portfolio based on June 30, 2022, return forecasts. In this 12-month period, there were significant changes in market conditions, most notably a large increase in bond yields, which in turn caused a significant upward shift in our VCMM return expectations for bonds.<sup>2</sup> As a result, the optimal after-tax allocation also changed, shifting away from equities toward bonds.

While we would not expect significant portfolio shifts to occur often, given how different portfolio allocations can be based on initial conditions, it is prudent to recalibrate the portfolio when there are material changes to capital market expectations. This model-based, rigorous, and holistic approach to constructing tax-aware portfolios gives us precisely this capability.

**FIGURE 7.**  
**Time-varying asset allocation assuming the highest federal tax rates**



**Notes:** The portfolios assume the highest tax rates of 20% for dividends and capital gains and 37% for income. Portfolios are constructed based on 10-year VCMM asset class forecasts, as of June 30, 2022. See Appendix B for more information on the VCMM.

**Source:** Vanguard.

<sup>2</sup> See Appendix C for the VCMM total and income return forecasts.



## Other use cases for the new methodology

The main objective of our tax-aware VAAM methodology is to quantify the trade-offs between stocks and bonds, as well as taxable and tax-exempt bonds, for taxable accounts, primarily for high-net-worth investors. The analysis above demonstrates what these trade-offs are and how those decisions are made; however, there are other use cases for this methodology, to assist in portfolio decision-making and to help improve after-tax investment outcomes for investors.

### An investment portfolio as the sole source of income or the source of additional income

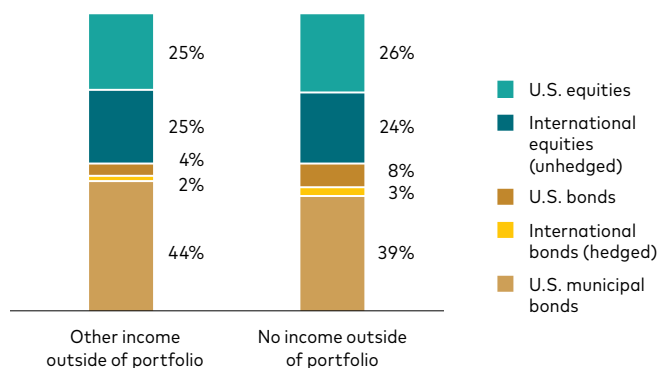
In the above analysis, flat tax rates were applied to the entire portfolio. The results can be very helpful for investors whose investment portfolio does not represent their entire income. However, for investors who are relying on their investment portfolio for income, constructing the portfolio based on their expected income level is more accurate.

This is accomplished by weighting the asset allocation within the portfolio for each combination of income, dividend, and capital gains taxes, up to the total income level. For example, as of 2022, a household that files jointly with an income of \$50,000 would pay an income tax of 10% on the first \$20,550 of income and of 12% on the remaining income. There is no dividend and capital gains tax for income under \$83,550.

For this client, an optimal after-tax portfolio would be a blend with 41% allocated to a portfolio with a 0% dividend and capital gains tax and 10% income tax (on the first \$20,550), and the remaining 59% allocated to a portfolio with a 0% dividend and capital gains tax and 12% income tax (on the remaining income). This process for blending portfolios can be done for any income level, in effect using the marginal tax rates to create optimal after-tax portfolios.

Figure 8 shows the difference in portfolio allocations for two portfolios, one for investors with other sources of income and the other for investors who rely on the portfolio as their sole source of income. The first portfolio assumes the highest level of taxation: 20% dividend and capital gains tax and 37% interest income tax. The second portfolio assumes a target portfolio income of \$200,000, so a portion of the dividend and capital gains would not be taxed, another portion would be taxed at 15%, and a portion of interest income would be taxed at 10%, 12%, 22%, and 24%, based on the appropriate income tax bracket. The result is a portfolio that has a lower municipal bond allocation in favor of taxable bonds, since the first portion of the taxable bond income is taxed at much lower rates.

**FIGURE 8.**  
**Impact of an investor's source of income**



**Notes:** Portfolios are constructed based on 10-year VCMM asset class forecasts, as of June 30, 2022. See Appendix B for more information on the VCMM.

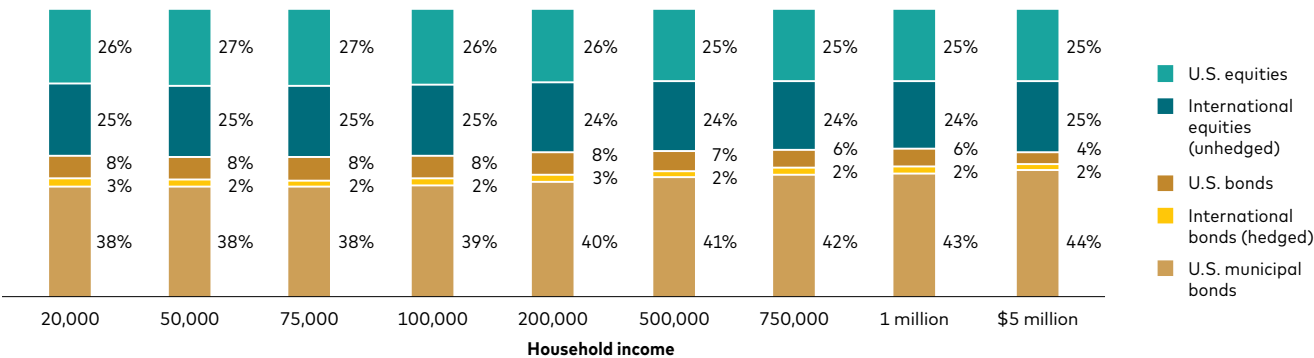
**Source:** Vanguard.

### Constructing a portfolio for different income levels

The typical ad hoc portfolio that uses municipal bonds for the entire bond portion is agnostic when it comes to an investor's income level and assumes the highest income tax bracket. Along with the ability to optimize the portfolio allocation on an after-tax basis, the tax-aware VAAM can differentiate based on income level by applying different tax rate assumptions. This is accomplished using the income bracket approach described above, where we assume an investor's entire income is generated by the portfolio.

Figure 9 compares optimal portfolio allocations based on a wide range of income levels, assuming a married household and filing jointly. To achieve optimal after-tax outcomes, lower-income households get lower allocations to municipal bonds and higher allocations to taxable bonds than higher-income households do. This demonstrates how the methodology can be scaled to build customized, and more precise, portfolio solutions for a variety of clients.

**FIGURE 9.**  
**Optimal allocations for various household income levels, using marginal tax rates**



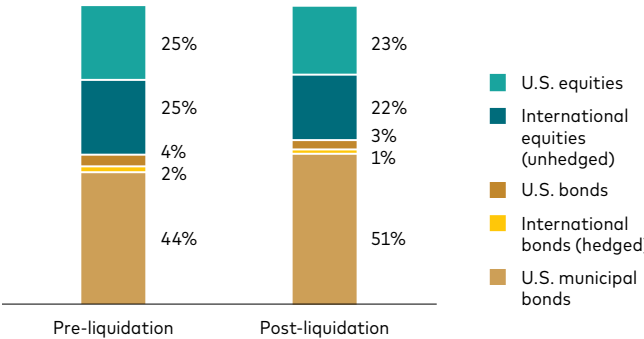
**Notes:** Portfolios are constructed based on 10-year VCMM asset class forecasts, as of June 30, 2022. See Appendix B for more information on the VCMM. Percentage totals may not equal 100 because of rounding.

**Source:** Vanguard.

**End-of-period hold or liquidation strategies**

With our enhanced methodology, we add the ability to optimize a portfolio based on whether the investor intends to remain invested in the portfolio or to liquidate it at the end of the period. This decision could greatly affect the asset allocation of the portfolio. In **Figure 10**, we compare the optimal allocations for before and after a liquidation. Equities typically accumulate much larger capital gains than bonds. Because of this, a portfolio that is to be liquidated at the end of the investment period has a much lower allocation to equities, in order to lower the capital gains tax burden after liquidation, and therefore improving the after-tax return outcomes.

**FIGURE 10.**  
**Assuming that a portfolio will be liquidated has an impact on asset allocation**



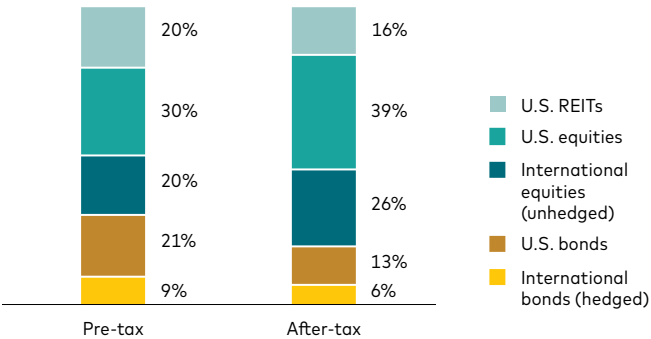
**Notes:** The portfolios assume the highest tax rates of 20% for dividends and capital gains and 37% for income. Portfolios are constructed based on 10-year VCMM asset class forecasts, as of June 30, 2022. See Appendix B for more information on the VCMM.  
**Source:** Vanguard.

**Optimizing a portfolio with tax-inefficient investments**

Many clients have expressed interest in a dedicated exposure to real estate investment trusts (REITs) in their portfolio, or to some other income-producing asset, that may not necessarily be tax efficient. The tax-aware VAAM provides the ability to optimize a portfolio using these assets, but now on an after-tax basis.

**Figure 11** compares the allocations of a strategic portfolio that includes REITs. REIT dividends are taxed at ordinary income tax rates up to the maximum rate of 37%, just like taxable bonds. And along with taxable bonds, the allocation to REITs is lower for an after-tax optimized portfolio than for a tax-agnostic portfolio, in favor of broad U.S. and international equities.

**FIGURE 11.**  
**Allocation among so-called tax-inefficient investments**



**Notes:** The after-tax portfolio assumes the highest tax rates of 20% for dividends and capital gains and 37% for income. Portfolios are constructed based on 10-year VCMM asset class forecasts, as of June 30, 2022. See Appendix B for more information on the VCMM.  
**Source:** Vanguard.

## Measuring the benefit of the new methodology

As we described above, in constructing tax-efficient portfolios, the industry standard practice is to substitute municipal bonds in place of taxable bonds. This ad hoc approach produces portfolios that are less diversified, add additional risk into the portfolio in the form of credit risk, and don't consider existing market conditions. It also does not account for an investor's income level, or whether the portfolio is the investor's sole source of income or is used for additional income, factors that can have meaningful implications for a portfolio.

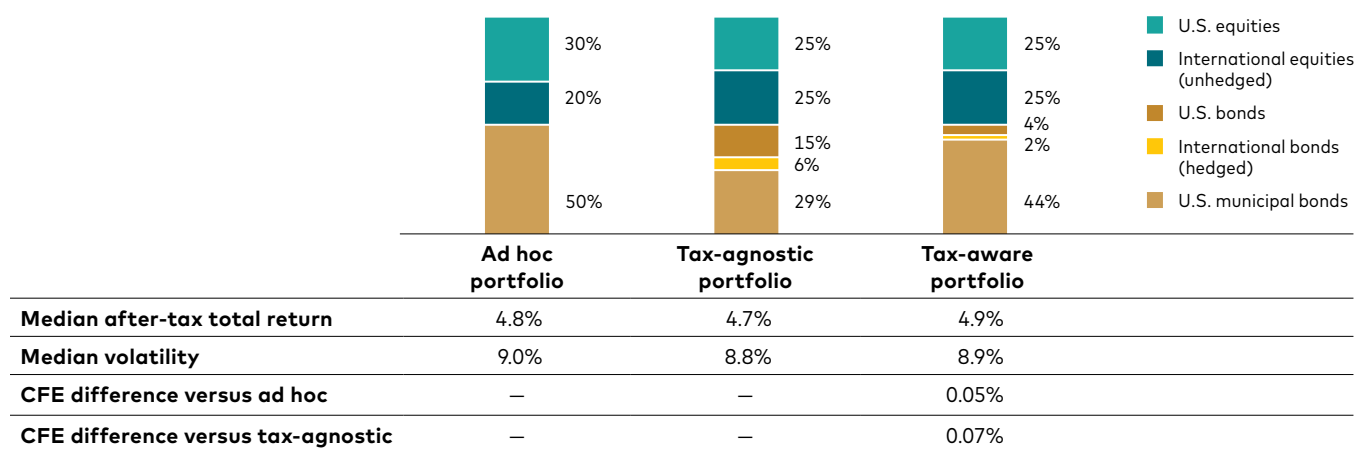
The tax-aware VAAM methodology can take the above inputs and optimize around them. To show the benefit over the ad hoc approach or the traditional tax-agnostic approach, the after-tax portfolio returns and volatilities can be contrasted for taxable accounts. In addition, the efficiency of the portfolios can be quantified by comparing their utility scores using a concept of certainty-

fee equivalent (CFE), or the additional cost an investor should be willing to pay to invest in one portfolio relative to another.<sup>3</sup> The higher the CFE, the greater the benefit.

**Figure 12** displays three portfolios and the associated metrics that were derived using 10-year VCMM capital market forecasts, on an after-tax basis, as of June 30, 2022. For this example, a flat tax rate of 20% is assumed for dividend and capital gains, and a 37% flat tax rate is assumed for interest income. The first portfolio is a typical static 50/50 portfolio, consisting of 50% stocks (60% U.S. and 40% international) and 50% bonds with the ad hoc substitution of municipal bonds for the entire portion of the bond allocation. The second portfolio is a tax-agnostic VAAM-derived portfolio. Finally, the last portfolio is a tax-aware VAAM portfolio, incorporating our 10-year forecasts on an after-tax basis, discounting the sub-asset-class returns by their appropriate tax rates, and taxing capital gains during annual rebalancing.

**FIGURE 12.**  
**Portfolio analytics**

As of June 30, 2022



**Notes:** Portfolio performance, median volatility, and CFE are based on 10-year VCMM asset class forecasts, as of June 30, 2022. Portfolio expectations are represented by the median simulation from the VCMM.

**Source:** Vanguard.

**IMPORTANT:** The projections and other information generated by the Vanguard Capital Markets Model regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. See Appendix B for more information.

**3** CFE is calculated as the difference between an existing portfolio and the VAAM optimal portfolio based on the latest inputs and capital market assumptions. It can be taken as the additional fee (in basis points) that an investor is willing to pay to move from an existing portfolio to one that is optimized. The CFE calculation is derived from the utility value attached to a certain portfolio risk-return trade-off, as expressed through the coefficient of risk aversion in utility functions.

As of June 30, 2022, the ad hoc portfolio was projected to produce an after-tax total return of 4.8%, annualized over 10 years, with a median volatility of 9.0%.

Using the traditional VAAM, we can see that a 50/50 allocation produces a portfolio that is much more diversified, including both U.S. and non-U.S. taxable bonds. Because the VAAM is tax agnostic, the decision to include the additional asset classes is based on the return distributions of expected total returns, correlations, and volatilities. When we use these allocations, and then apply taxes to the return expectations, this portfolio yields a 10-year median annualized after-tax return of 4.7%, with a median volatility of 8.8%.

Finally, the third portfolio, which was derived using the new tax-aware methodology, has a lower allocation to taxable bonds and a higher

one to municipal bonds. The allocation to taxable bonds is lower because they have worse expected risk-adjusted returns over the next 10 years, combined with a very high tax burden (37%) on their income return. With all these inputs accounted for, this portfolio is expected to produce slightly better risk-adjusted after-tax returns, with a median after-tax return of 4.9% and a median expected volatility of 8.9%.

Another way to quantify how much more efficient one portfolio is relative to another is to compare utility scores for the portfolios and then compute the CFE. The higher the CFE, the better the expected risk-return trade-off. Using this metric, we find that as of June 30, 2022, the CFE for the optimal after-tax VAAM portfolio was 5 basis points higher than for a typical ad hoc portfolio, and 7 basis points greater than for the tax-agnostic portfolio.

## Caveats

In this paper we have presented our new framework for optimizing portfolio allocations on an after-tax basis. We have tested the methodology by applying commonly used inputs and assumptions, such as federal tax rates for dividends and capital gains, and income tax rates. With this approach, we are confident that we can account for a large portion of client scenarios and optimize portfolios. There are, however, several other considerations that we have not yet tested or evaluated, including alternative minimum tax, treatment of nonqualified dividends for equities, and rebalancing based on specific tax lots, which we have omitted for the sake of simplicity in this analysis. While we expect these additional considerations to result in only small changes in optimal allocations, we nonetheless intend to conduct further research to incorporate them as we continue enhancing the methodology.

## Conclusion

This paper introduces enhancements to make the VAAM tax-aware, by accounting for taxes on equity dividends, bond interest, and capital gains due to rebalancing, as well as current market conditions. This model-based approach has shown that the traditional ad hoc substitution approach may not be optimal. Given asset class projections over the next 10 years, many investors could benefit from keeping some portion of their bond allocation in taxable bonds, even after accounting for taxes.

This new methodology adds the capability to optimize portfolios based on maximizing the utility of wealth on an after-tax basis, for various client scenarios. Using this methodology, investors can construct portfolios that better reflect the after-tax trade-offs between stocks and bonds and between taxable and municipal

bonds. The tax-aware VAAM doesn't just optimize a portfolio based on the tax efficiency of an asset class; it also considers the riskiness of the asset, as in the case with municipal bonds and the additional credit risk they introduce into the portfolio.

To help achieve better outcomes over the medium term (10 years), the tax-aware VAAM can adjust portfolios when market conditions change significantly. The enhancement also provides the ability to build portfolios for other use cases, most notably to account for whether the portfolio is expected to generate all of an investor's income or only a portion of it. Additionally, the tax-aware VAAM can account for different income levels and tax rates, for whether the portfolio is to be liquidated at the end of the period, and for the use of tax-inefficient investments, such as REITs. As illustrated in the paper, when these underlying assumptions and conditions change, optimal portfolios can be quite different from industry norms.

## References

Aliaga-Díaz, Roger, Harshdeep Ahluwalia, Giulio Renzi-Ricci, Todd Schlanger, Victor Zhu, and Carole Okigbo, 2022. *Vanguard's Portfolio Construction Framework: From Investing Principles to Custom Portfolio Solutions*. Valley Forge, Pa.: The Vanguard Group.

Aliaga-Díaz, Roger, Giulio Renzi-Ricci, Ankul Daga, and Harshdeep Ahluwalia, 2019. *Vanguard Asset Allocation Model: An Investment Solution for Active-Passive-Factor Portfolios*. Valley Forge, Pa.: The Vanguard Group.

Wallick, Daniel W., Kevin DiCiurcio, Darrell I. Pacheco, and Roger Aliaga-Díaz, 2020. *The Implications of Time-Varying Return on Portfolio Construction*. Valley Forge, Pa.: The Vanguard Group.

## Appendix A: The Vanguard Asset Allocation Model

The Vanguard Asset Allocation Model (VAAM) aims to determine the optimal asset allocation for an investor based on two key inputs: the investor's attitude toward the trade-off between risk and return, and the output of Vanguard's VCMM model. The former is estimated using a power utility function that aims to capture the investor's subjective perception of the relationship between risk and return through the risk aversion coefficient:

$$U(W) = \begin{cases} W^{1-\gamma}, & \gamma < 1 \\ \ln(W), & \gamma = 1 \end{cases}$$

where  $\gamma$  is the relative risk aversion (RRA) coefficient and  $W$  is the level of terminal wealth relative to starting wealth.  $W$  will compound in each time period by the total multiasset portfolio return  $R_t$ :

$$R_t = \sum_{i=1}^N x_i r_{i,t} = \sum_{i=1}^N x_i^p r_{i,t}^p + \sum_{i=1}^N \sum_{f=1}^F x_i^f r_{i,t}^f + \sum_{i=1}^N x_i^a r_{i,t}^a$$

$$\begin{cases} r_{i,t}^p = r_{i,t}^M \\ r_{i,t}^f = r_{i,t}^M + \delta_{i,t}^f \\ r_{i,t}^a = a_i + \beta_i r_{i,t}^M + \sum_{f=1}^F L_i^f \delta_{i,t}^f + \varepsilon_{i,t} \end{cases} \quad \varepsilon_{i,t} \sim t(\nu) \sqrt{\sigma_{a_i}^2}$$

where  $x_i$  and  $r_i$  are the portfolio weights and relative total returns for each asset class  $i$  and superscripts  $p$ ,  $f$ , and  $a$  refer to passive, factors,

and active, respectively. The market benchmark return is represented by  $r_{i,t}^M$ ,  $\delta_{i,t}^f$  is the excess (to the market benchmark) factor return for factor  $f$ , and  $\beta_i$  and  $L_i$  correspond to the market beta and factor loading for each asset class, respectively.  $\alpha_i$  is the factor-adjusted excess active return.

The portfolio choice problem consists then of finding optimal weights for each passive asset class, factor, or active manager or strategy in the portfolio. Weights are selected to maximize the expected utility of final portfolio wealth, expressed as:

$$\max_w \mathbb{E} [U(W_T)] \rightarrow \max_w \left\{ \mathbb{E} \left[ \frac{W_p^{1-\gamma_p}}{1-\gamma_p} \right] + \mathbb{E} \left[ \frac{W_f^{1-\gamma_f}}{1-\gamma_f} \right] + \mathbb{E} \left[ \frac{W_a^{1-\gamma_a}}{1-\gamma_a} \right] \right\}$$

$$\text{s.t. } \{w_i \in \mathbb{R} \mid 0 \leq w_i \leq 1\} \wedge \sum_i w_i = 1$$

$$\sum_i C \cdot w_i \leq b$$

where  $W_p$ ,  $W_f$ , and  $W_a$  are the wealth at maturity coming from systematic, factor, and factor-adjusted alpha exposures, respectively,  $\gamma_p$ ,  $\gamma_f$ , and  $\gamma_a$  are the systematic, factor, and alpha risk aversions, respectively, and  $C$  and  $b$  refer to the set of linear inequality constraints.

## Appendix B: The Vanguard Capital Markets Model

**IMPORTANT: The projections and other information generated by the Vanguard Capital Markets Model (VCMM) regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. VCMM results will vary with each use and over time.**

The VCMM projections are based on a statistical analysis of historical data. Future returns may behave differently from the historical patterns captured in the VCMM. More important, the VCMM may be underestimating extreme negative scenarios unobserved in the historical period on which the model estimation is based.

The VCMM is a proprietary financial simulation tool developed and maintained by Vanguard's primary investment research and advice teams. The model forecasts distributions of future returns for a wide array of broad asset classes. Those asset classes include U.S. and international equity markets, several maturities of the U.S.

Treasury and corporate fixed income markets, international fixed income markets, U.S. money markets, commodities, and certain alternative investment strategies. The theoretical and empirical foundation for the VCMM is that the returns of various asset classes reflect the compensation investors require for bearing different types of systematic risk (beta). At the core of the model are estimates of the dynamic statistical relationship between risk factors and asset returns, obtained from statistical analysis based on available monthly financial and economic data from as early as 1960. Using a system of estimated equations, the model then applies a Monte Carlo simulation method to project the estimated interrelationships among risk factors and asset classes as well as uncertainty and randomness over time. The model generates a large set of simulated outcomes for each asset class over several time horizons. Forecasts are obtained by computing measures of central tendency in these simulations. Results produced by the tool will vary with each use and over time.



## Appendix C: Data tables

**FIGURE C-1.**  
**VCMM return distributions**

VCMM 10-year return distributions as of June 30, 2021

		Asset class					
		U.S. equities	International equities	U.S. bonds	International bonds	Municipal bonds	REITs
Total return	5th percentile	-3.4%	-0.1%	0.7%	0.2%	0.3%	-5.1%
	25th percentile	0.6	3.7	1.4	1.0	1.5	-0.2
	50th percentile	3.3	6.2	1.9	1.8	2.3	3.1
	75th percentile	6.0	8.8	2.5	2.7	3.0	6.5
	95th percentile	10.0	12.6	3.4	4.1	4.3	11.6
	Median volatility	16.7	18.4	4.6	3.8	4.7	19.4
Income return	5th percentile	0.9	2.5	1.0	0.1	0.9	1.0
	25th percentile	1.2	2.8	1.8	0.5	1.5	1.9
	50th percentile	1.5	3.1	2.5	0.9	2.2	2.6
	75th percentile	1.8	3.4	3.3	1.3	2.9	3.4
	95th percentile	2.4	3.9	4.6	1.8	4.0	4.6
	Median volatility	0.3	0.5	0.9	0.5	0.8	1.2

VCMM 10-year return distributions as of June 30, 2022

		Asset class					
		U.S. equities	International equities	U.S. bonds	International bonds	Municipal bonds	REITs
Total return	5th percentile	-2.1%	1.4%	2.2%	1.5%	1.8%	-4.3%
	25th percentile	2.1	5.0	3.0	2.6	3.0	0.5
	50th percentile	5.1	7.6	3.7	3.5	3.9	3.8
	75th percentile	8.1	10.2	4.3	4.5	4.7	7.2
	95th percentile	12.5	14.0	5.4	6.0	6.0	12.3
	Median volatility	17.6	18.6	5.1	4.3	5.1	19.9
Income return	5th percentile	1.1	2.6	1.8	1.2	1.5	1.5
	25th percentile	1.5	3.0	2.9	1.7	2.4	2.7
	50th percentile	1.8	3.3	3.7	2.1	3.2	3.5
	75th percentile	2.2	3.6	4.5	2.4	3.9	4.4
	95th percentile	3.1	4.2	5.8	3.0	5.1	5.8
	Median volatility	0.3	0.4	1.0	0.5	0.9	1.4

**Notes:** The forecast corresponds to the distribution of 10,000 VCMM simulations for 10-year annualized nominal returns in USD for asset classes highlighted here. Median volatility is the 50th percentile of an asset class's distribution of annualized standard deviation of returns. Asset class returns do not take into account management fees and expenses, nor do they reflect the effect of taxes. Returns do reflect reinvestment of dividends and capital gains. Indexes are unmanaged; therefore, direct investment is not possible.

**Source:** Vanguard.

**IMPORTANT:** The projections and other information generated by the Vanguard Capital Markets Model regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results.

FIGURE C-2.

**2022 tax brackets**

2022 interest income / nonqualified dividend / ordinary income tax rates

Rate	Single	Married filing jointly	Married filing separately	Head of household
<b>10%</b>	\$0–\$10,275	\$0–\$20,550	\$0–\$10,275	\$0–\$14,650
<b>12%</b>	\$10,276–\$41,775	\$20,551–\$83,550	\$10,276–\$41,775	\$14,651–\$55,900
<b>22%</b>	\$41,776–\$89,075	\$83,551–\$178,150	\$41,776–\$89,075	\$55,901–\$89,050
<b>24%</b>	\$89,076 –\$170,050	\$178,151–\$340,100	\$89,076–\$170,050	\$89,051–\$170,050
<b>32%</b>	\$170,051–\$215,950	\$340,101–\$431,900	\$170,051–\$215,950	\$170,051–\$215,950
<b>35%</b>	\$215,951–\$539,900	\$431,901–\$647,850	\$215,951–\$539,900	\$215,951–\$539,900
<b>37%</b>	\$539,901+	\$647,851+	\$539,901+	\$539,901+

2022 qualified dividend / long-term capital gains tax rates

Rate	Single	Married filing jointly	Married filing separately	Head of household
<b>0%</b>	\$0–\$41,675	\$0–\$83,350	\$0–\$41,675	\$0–\$55,800
<b>15%</b>	\$41,676–\$459,750	\$83,351–\$517,200	\$41,676–\$258,600	\$55,801–\$488,500
<b>20%</b>	\$459,751+	\$517,201+	\$258,601+	\$488,501+

Source: IRS.

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